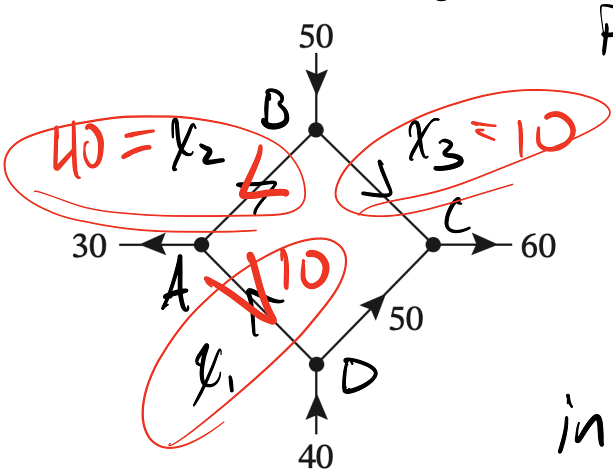
1.10 – Applications of Linear Systems

Definitions: A **network** is a set of branches through which something flows. Branches meet at **nodes** or **junctions**. Networks in our work will feature **flow conservation**, meaning the rate of flow in and out are equal at each node.

#1 The accompanying figure shows a network in which the flow rate and direction of flow in certain branches are known. Find the flow rates and directions of flow in the remaining branches.



Pick a direction

rate in = rate out

in   out

Node A: $x_1 = 30 + x_2$ $x_1 - x_2 = 30$

Node B: $x_2 + 50 = x_3$ $\rightarrow$ $x_2 - x_3 = -50$

Node C: $x_3 + 50 = 60$ $x_3 = 10$

Solving: Use an appropriate method.
(ideally practicing methods from this corse,
but honestly in this case, it's overkill)

$x_3 = 10 \Rightarrow x_2 - 10 = -50 \Rightarrow x_2 = -40$

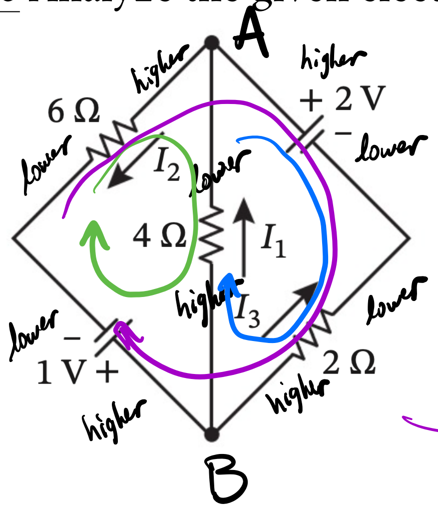
$x_1 + 40 = 30 \Rightarrow x_1 = -10$

Definitions: A **battery** is a source of electric energy, and a **resistor** dissipates electric energy. A **node** is where three or more wires join in a circuit. A **closed loop** begins and ends at the same node.

Ohm's Law: $E = IR$, where E (volts) is voltage drop at a resistor with resistance R (ohms) in a circuit with current I (amperes).

Kirchhoff's Laws (summary): Net current (in and out) at a node is zero, and net voltage change (rises and drops) in a closed loop is zero.

#6 Analyze the given electrical circuit by finding the unknown currents.



Nodes   in(pos)   out(neg)

Node A: $I_1 + I_3 - I_2 = 0$

Node B: $I_2 - I_1 - I_3 = 0$

equivalent equations

→ Outer loop: $2I_3 + 6I_2 - 1 - 2 = 0$ (redundant)

Clockwise right loop

Voltage rises   voltage drops

$2I_3 \quad -4I_1 \quad -2 \quad = 0$

CW left loop

$6I_2 + 4I_1 \quad -1 \quad = 0$

System: $I_1 - I_2 + I_3 = 0$ (Node A)

$-4I_1 + 2I_3 = 2 \rightarrow -2I_1 + I_3 = 1$

$4I_1 + 6I_2 = 1$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ -2 & 0 & 1 & 1 \\ 4 & 6 & 0 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 0 & 0 & -5/22 \\ 0 & 1 & 0 & 7/22 \\ 0 & 0 & 1 & 6/11 \end{array}\right]$$

$$I_1 = -\frac{5}{22}\,A, \quad I_2 = \frac{7}{22}\,A, \quad I_3 = \frac{6}{11}\,A$$

The arrow given for I_1 points in the wrong direction

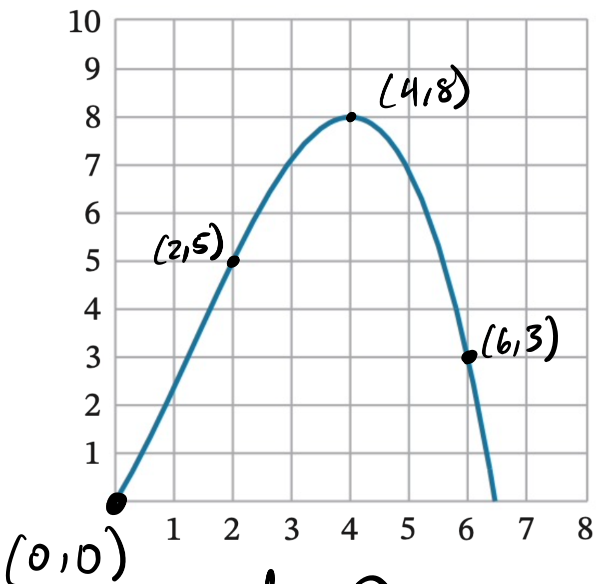
Theorem 1.10.1 Polynomial Interpolation
Given any n points in the xy-plane that have distinct x-coordinates, there is a unique polynomial of degree $n-1$ or less whose graph passes through those points.

#16 The accompanying figure shows the graph of a cubic polynomial. Find the polynomial.

$P(x) = \underline{a}x^3 + \underline{b}x^2 + \underline{c}x + \underline{d}$

(General cubic polynomial.

For our augmented matrix setup, use $P(x) = d + cx + bx^2 + ax^3$



$\rightarrow d = 0$

$(0,0)$ $0 = d + 0c + b0^2 + a0^3$

$(2,5)$ $5 = d + 2c + 4b + 8a$

$(4,8)$ $8 = d + 4c + 16b + 64a$

$(6,3)$ $3 = d + 6c + 36b + 216a$

$$\begin{array}{cccc|c} \cancel{d} & c & b & a & \\ \cancel{a} & \cancel{b} & \cancel{c} & \cancel{d} & y \end{array}$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 8 & 5 \\ 4 & 16 & 64 & 8 \\ 6 & 36 & 216 & 3 \end{array}\right] \xrightarrow[R_3 \to \frac{1}{3}R_3]{R_2 \to \frac{1}{4}R_2} \left[\begin{array}{ccc|c} 2 & 4 & 8 & 5 \\ 1 & 4 & 16 & 2 \\ 2 & 12 & 72 & 1 \end{array}\right] \quad R_2 \leftrightarrow R_1$$

$$\left[\begin{array}{ccc|c} 1 & 4 & 16 & 2 \\ 2 & 4 & 8 & 5 \\ 2 & 12 & 72 & 1 \end{array}\right] \rightarrow \left[\begin{array}{ccc|c} 1 & 4 & 16 & 2 \\ 0 & -4 & -24 & 1 \\ 0 & 4 & 40 & -3 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$

$$\begin{array}{cccc} 2 & 4 & 8 & 5 \\ -2 & -8 & -32 & -4 \\ \hline 0 & -4 & -24 & 1 \end{array}$$

$R_3 \to R_3 - 2R_1$

$$\begin{array}{cccc} 2 & 12 & 72 & 1 \\ -2 & -8 & -32 & -4 \\ \hline 0 & 4 & 40 & -3 \end{array}$$

$R_3 \to R_3 + R_2$

$$\begin{array}{cccc} 0 & 4 & 40 & -3 \\ 0 & -4 & -24 & 1 \\ \hline 0 & 0 & 16 & -2 \end{array}$$

$$\begin{array}{cccc} c & b & a & \\ & \left[\begin{array}{ccc|c} 1 & 4 & 16 & 2 \\ 0 & -4 & -24 & 1 \\ 0 & 0 & 1 & -1/8 \end{array}\right] & & \end{array}$$

$\rightarrow -4b - 24\left(-\frac{1}{8}\right) = 1 \Rightarrow b = \frac{1}{2}$

$\rightarrow a = -\frac{1}{8}$

$\rightarrow c + 4\left(\frac{1}{2}\right) + 16\left(-\frac{1}{8}\right) = 2$

$c + 2 - 2 = 2$

Recall $d = 0$

$$p(x) = -\frac{1}{8}x^3 + \frac{1}{2}x^2 + 2x$$